P425/1
PURE MATHEMATICS
Paper 1
3 hours

WAKISSHA

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer* **all** *the eight questions in section* **A** *and any* **five** *questions from section* **B**.
- *Any additional question(s) answered will* **not** *be marked.*
- *Show* **all** *necessary working clearly.*
- *Begin each answer on a fresh page of paper.*
- *Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.*

Turn Over

SECTION A (40 MARKS)

Answer **all** questions in this section.

1. Solve the simultaneous equations.
   $p + q + r = 0$, $p + 2q + 2r = 2$ and $2p + 3r = 4$. (05marks)

2. Determine the Cartesian's equation of a line passing through points A (2,5,4) and B (5,3,7) (05marks)

3. A Circle with Centre C, cuts another circle $x^2 + y^2 - 4x + 6y - 7 = 0$ at right angles and passes through the point (1, 3). Find the locus of Centre C. (05marks)

4. Solve the equation $\tan^{-1}(2x+1) = \tan^{-1}(2) - \tan^{-1}(2x-1)$ (05marks)

5. Evaluate $\int_0^{\sqrt{\frac{\pi}{2}}} \frac{x}{1+\sin(x^2)}dx$ (06marks)

6. A committee of four pupils is to be selected from three boys and seven girls. How many committees are formed in order to have girls as the majority in committee? (04marks)

7. Use Maclaurin's theorem to expand $In\sqrt{\left(\frac{1+2x}{1+x}\right)}$ up to x^2 (05marks)

8. The inside of a glass is in the shape of an inverted cone of depth 8cm and radius 4cm full of wine. The wine is leaking from small hole at vertex at rate $0.06cm^3s^{-1}$ into somebody mouth. Find the rate at which surface area of wine in contact with glass in decreasing when depth is 6cm. (05marks)

SECTION B (60 marks)

Answer any **five** questions from this section.

9. (a) Solve inequality; $\frac{x-2}{x+1} \geq \frac{x+1}{x+3}$ (06marks)

   (b) John deposits Shs. 3,000,000 at beginning of every year in a micro-finance bank starting 2015, how much would he collect at the end of 2020 if the bank offers compound interest of 12.5% per annum and no withdrawal is made within the period. (06marks)

10. (a) Find the vector equation of the line passing through the point (3,1, 2) and perpendicular to the plane $r.(2i - j + k) = 4$ hence find point of intersection of line and the plane. (06marks)

(b) The position vectors of the points A, B and C are $2i - j + 5k, i - 2j + k$ and $3i + j - 2k$ respectively. Given that L and M are mid-points of AC and CB respectively.
Show that BA = 2ML (06marks)

11. (a) Solve $\text{Cos}3\theta + \text{Cos}2\theta + \text{Cos}\theta = 0, 0^0 \leq \theta \leq 180^0$ (05marks)

(b) Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$, hence find all solutions of the equation $8x^3 - 6x + 1 = 0$. Correct to 3 decimal places. (07marks)

12. Given curve $y = \frac{(x-3)^2}{(x-9)(x-1)}$. Find equations of asymptotes and sketch the curve. (12marks)

13. Express $f(x) = \frac{x^3 + 4x^2 - 5x - 4}{(x-2)^2(1+x^2)}$ into partial fractions, hence evaluate $\int_3^5 f(x)dx$.

(12marks)

14. (a) Solve $x\frac{dy}{dx} + 2y = x^2$ when y (1) = 1. (05marks)

(b) A liquid cools in the environment of a constant temperature of 21^0C at the rate proportional to the excess temperature. Initially the temperature of liquid is 100^0C and after 10 minutes the temperature dropped by 16^0C. Find how long it takes for the temperature of liquid to be 70^0C. (07marks)

15. (a) Given that the root of $z^4 - 4z^3 + 3z^2 + 3z^2 2z - 6 = 0$ is 1-i, find other roots. (06marks)

(b) Evaluate $\left(1 + i\sqrt{3}\right)^{\frac{2}{3}}$ (06marks)

16. (a) Find the equation of a circle which is a tangent to the lines $3y = 4x$, $y = 8$ and $4x + 3y = 0$ (05marks)

(b) If the line $y = mx + c$ is a tangent to the ellipse $a^2y^2 + b^2x^2 = a^2b^2$, prove that $C^2 = b^2 + a^2m^2$. Hence determine the equations of the common tangents to ellipse $4x^2 + 14y = 56$ and $3x^2 + 23y^2 = 69$ (07marks)

END